Section 6.1 The Greatest Common Factor and Factoring by Grouping

Greatest Common Factor (GCF):

- The GCF is an expression of the highest degree that divides each term of the polynomial.
- The variable part of the greatest common factor always contains **the smallest power** of a variable that appears in all terms of the polynomial.

Finding GCF

- Step 1: Find the prime factorization of all integers and integer coefficients
- Step 2: List all the factors that are common to all terms, including variables
- Step 3: Choose the smallest power for each factor that is common to all terms
- Step 4: Multiply these powers to find the GCF
  NOTE: If there is no common prime factor or variable, then the GCF is 1

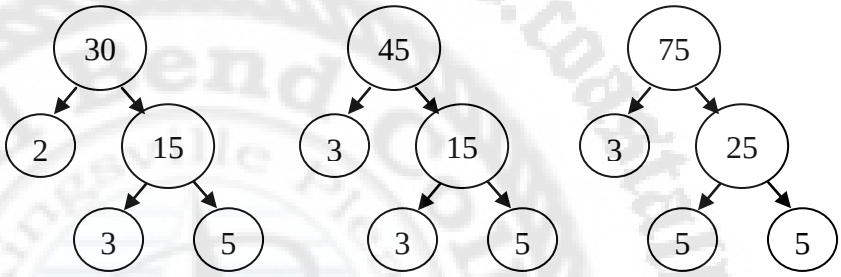
(Ex) Find the GCF for the following set of algebraic terms: $\{30x^4y, 45x^3y, 75x^5y^2\}$

$30x^4y = 2\cdot\underline{\mathbf{3}}\cdot\underline{\mathbf{5}}\cdot\underline{\boldsymbol{x\cdot x\cdot x}}\cdot x\cdot\underline{\boldsymbol{y}}$

$45x^3y = 3\cdot\underline{\mathbf{3}}\cdot\underline{\mathbf{5}}\cdot\underline{\boldsymbol{x\cdot x\cdot x}}\cdot\underline{\boldsymbol{y}}$

$75x^5y^2 = \underline{\mathbf{3}}\cdot\underline{\mathbf{5}}\cdot 5\cdot\underline{\boldsymbol{x\cdot x\cdot x}}\cdot x\cdot x\cdot\underline{\boldsymbol{y}}\cdot y$

Therefore, GCF is $15x^3y$



¿Negative GCF?

- In my opinion, we do not need to factor out a negative GCF
- Factoring out a negative GCF is exactly same as factoring out GCF, but last step.
  - Step 1: Find the prime factorization of all integers and integer coefficients
  - Step 2: List all the factors that are common to all terms, including variables
  - Step 3: Choose the smallest power of each factor that is common to all terms
  - Step 4: Multiply these powers to find the GCF
  - Step 5: Put negative sign in front of the GCF

Factoring by Grouping will be discussed with examples

Excercises

Find the greatest common factor of the following list of terms. $18y,\ 8y^4,$ and $-10y^5$ The greatest common factor is ☐.	(Solution 1) Step 1: Prime Factorization (ignore signs) $18y = 2\cdot 3\cdot 3y = 2\cdot 3^2y$ $8y^4 = 2\cdot 2\cdot 2y^4 = 2^3y^4$ $10y^5 = 2\cdot 5y^5 = 2\cdot 5y^5$ Step 2: List all factors that are common to all terms Common factors to all are 2 and y Step 3: Choose the smallest power The smallest power for 2 is 1 The smallest power for y is 1 So, the GCF is $2^1\cdot y^1 = 2y$

Section 6.1 The Greatest Common Factor and Factoring by Grouping

Factor the polynomial using the greatest common factor. If there is no common factor other than 1 and the polynomial cannot be factored, so state.

$6x^2 - 2x^4$

Select the correct choice below and fill in any answer boxes in your choice.

A. $6x^2 - 2x^4 =$ ☐

B. The polynomial cannot be factored.

(Solution 2)

Step 1: Prime Factorization (ignore signs)

$6x^2 = 2 \cdot 3x^2$

$2x^4 = 2 \cdot x^4$

Step 2: List all factors that are common to all terms

Common factors to all are 2 and x

Step 3: Choose the smallest power

The smallest power for 2 is 1

The smallest power for x is 2

So, the GCF is $2^1 \cdot x^2 = 2x^2$

Factoring out GCF

$\frac{6x^2}{2x^2} = 3 \Rightarrow 6x^2 = 2x^2 \cdot 3$

$\frac{2x^4}{2x^2} = x^2 \Rightarrow 2x^4 = 2x^2 \cdot x^2$

$6x^2 - 2x^4 = \mathbf{2x^2} \cdot 3 - \mathbf{2x^2} \cdot x^2$

$= \mathbf{2x^2}(3 - x^2)$

Factor the polynomial using the greatest common factor. If there is no common factor other than 1 and the polynomial cannot be factored, so state.

$7x^4 - 21x^3 + 35x^2$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $7x^4 - 21x^3 + 35x^2 =$ ☐ (Factor completely.)

B. The polynomial cannot be factored.

(Solution 3)

Step 1: Prime Factorization (ignore signs)

$7x^4 = 7x^4$

$21x^3 = 3 \cdot 7x^3$

$35x^2 = 5 \cdot 7x^2$

Step 2: List all factors that are common to all terms

Common factors to all are 7 and x.

Step 3: Choose the smallest power

The smallest power for 7 is 1

The smallest power for x is 2

So, the GCF is $7^1 \cdot x^2 = 7x^2$

Factoring out GCF

$\frac{7x^4}{7x^2} = x^2 \Rightarrow 7x^4 = 7x^2 \cdot x^2$

$\frac{21x^3}{7x^2} = 3x \Rightarrow 21x^3 = 7x^2 \cdot 3x$

$\frac{35x^2}{7x^2} = 5 \Rightarrow 35x^2 = 7x^2 \cdot 5$

$7x^4 - 21x^3 + 35x^2$

$= \mathbf{7x^2} \cdot x^2 - \mathbf{7x^2} \cdot 3x + \mathbf{7x^2} \cdot 5$

$= \mathbf{7x^2}(x^2 - 3x + 5)$

Factor the polynomial using the greatest common factor. If there is no common factor other than 1 and the polynomial cannot be factored, so state.

$27x^2 - 29$

Select the correct choice below and fill in any answer boxes within your choice.

A. $27x^2 - 29 =$ ☐

B. The polynomial cannot be factored.

(Solution 3)

Step 1: Prime Factorization (ignore signs)

$27x^2 = 3 \cdot 3 \cdot 3x^2 = 3^3x^2$

$29 = 29$ because 29 is prime number

Step 2: List all factors that are common to all terms

There is **no common factor**.

Therefore, $27x^2 - 29$ cannot be factored.

or $27x^2 - 29$ is prime.

Section 6.1 The Greatest Common Factor and Factoring by Grouping

Factor the following polynomial using the negative of the greatest common factor.

$-4x^4+24x^3+20x^2$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $-4x^4+24x^3+20x^2=$ ☐

B. The polynomial cannot be factored.

(Solution 5)

Step 1: Prime Factorization (ignore signs)

$4x^4 = 2\cdot 2x^4 = 2^2x^4$

$24x^3 = 2\cdot 2\cdot 2\cdot 3x^3 = 2^3\cdot 3x^3$

$20x^2 = 2\cdot 2\cdot 5x^2 = 2^2x^2$

Step 2: List all factors that are common to all terms

Common factors to all are 2 and x.

Step 3: Choose the smallest power

The smallest power for 2 is 2

The smallest power for x is 2

GCF= $2^2\cdot x^2 = 4x^2 \Longrightarrow$ the negative GCF is $-4x^2$

Factoring out the negative GCF

$\frac{-4x^4}{-4x^2} = x^2 \Longrightarrow -4x^4 = -4x^2\cdot x^2$

$\frac{24x^3}{-4x^2} = -6x \Longrightarrow 24x^3 = -4x^2\cdot(-6x)$

$\frac{20x^2}{-4x^2} = -5 \Longrightarrow 20x^2 = -4x^2\cdot(-5)$

$-4x^4+24x^3+20x^2$

$=\mathbf{-4x^2}\cdot x^2+(\mathbf{-4x^2})\cdot(-6x)+(\mathbf{-4x^2})\cdot(-5)$

$=\mathbf{-4x^2}(x^2-6x-5)$

Factor the following polynomial using the negative of the greatest common factor.

$-12a^3b^2+16ab$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $-12a^3b^2+16ab=$ ☐

B. The polynomial cannot be factored.

(Solution 6)

Step 1: Prime Factorization (ignore signs)

$-12a^3b^2 = 2\cdot 2\cdot 3a^3b^2 = 2^2\cdot 3a^3b^2$

$16ab = 2\cdot 2\cdot 2\cdot 2ab = 2^4ab$

Step 2: List all factors that are common to all terms

Common factors to all are 2, a, and b.

Step 3: Choose the smallest power

The smallest power for 2 is 2

The smallest power for a is 1

The smallest power for b is 1

GCF= $2^2ab = 4ab \Longrightarrow$ the negative GCF is $-4ab$

Factoring out the negative GCF

$\frac{-12a^3b^2}{-4ab} = 3a^2b^2 \Longrightarrow -12a^3b^2 = -4ab\cdot 3a^2b^2$

$\frac{16ab}{-4ab} = -4 \Longrightarrow 16ab = -4ab\cdot(-4)$

$-12a^3b^2+16ab$

$=(\mathbf{-4ab})(3a^2b)+(\mathbf{-4ab})(-4)$

$=(\mathbf{-4ab})(3a^2b-4)$

Section 6.1 The Greatest Common Factor and Factoring by Grouping

Factor the following polynomial using the greatest common binomial factor.

$12x(x+y)-(x+y)$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $12x(x+y)-(x+y)=$ ☐ (Factor completely.)

B. The polynomial cannot be factored.

(Solution 7)

Step 1: Prime Factorization (ignore signs)

$12x(x+y)=2\cdot 2\cdot 3x(x+y)=2^2\cdot 3x(x+y)$

$(x+y)=(x+y)$ since $(x+y)$ is prime

Step 2: List all factors that are common to all terms

Common factors to all are $(x+y)$.

Step 3: Choose the smallest power

The smallest power for $(x+y)$ is 1

So, the GCF is $(x+y)$

Factoring out GCF

$$\frac{12x(x+y)}{x+y}=12x;\quad \frac{-(x+y)}{x+y}=-1$$

$$12x(x+y)-(x+y)$$

$$=(\boldsymbol{x+y})(12x)+(\boldsymbol{x+y})(-1)$$

$$=(x+y)(12x-1)$$

Factor the following expression by grouping.

$x^2+8x+9x+72$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $x^2+8x+9x+72=$ ☐ (Factor completely.)

B. The polynomial cannot be factored.

(Solution 8)

Step 1: Count the number of terms and variables

$x^2+8x+9x+72$ has four terms and one variable x.

Step 2: Arrangement

Since $x^2+8x+9x+72$ has one variable, we arrange the expression in descending order.

$x^2+8x+9x+72$ is in descending order.

Step 3: Grouping by dividing terms evenly.

$x^2+8x+9x+72$ has four terms, so two terms in each group. $(x^2+8x)+(9x+72)$

Step 4: Factoring out GCF from each group

$$(x^2+8x)+(9x+72)$$

$$=x(\boldsymbol{x+8})+9(\boldsymbol{x+8})$$

$$=(\boldsymbol{x+8})(x+9)$$

Factor the following expression by grouping.

$x^3-7x^2+9x-63$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $x^3-7x^2+9x-63=$ ☐ (Factor completely.)

B. The polynomial cannot be factored.

(Solution 9)

Step 1: Count the number of terms and variables

$x^3-7x^2+9x-63$ has four terms and one variable x.

Step 2: Arrangement

Since $x^3-7x^2+9x-63$ has one variable, we arrange the expression in descending order.

$x^3-7x^2+9x-63$ is in descending order.

Step 3: Grouping by dividing terms evenly.

$x^3-7x^2+9x-63$ has four terms, so two terms in each group. $(x^3-7x^2)+(9x-63)$

Step 4: Factoring out GCF from each group

$$(x^3-7x^2)+(9x-63)$$

$$=x^2(\boldsymbol{x-7})+9(\boldsymbol{x-7})$$

$$=(\boldsymbol{x-7})(x^2+9)$$

Factor the following expression by grouping. $5x^3 - 2x^2 - 25x + 10$ Select the correct choice below and, if necessary, fill in the answer box to complete your choice. ○ A. $5x^3 - 2x^2 - 25x + 10 =$ ☐ (Factor completely.) ○ B. The polynomial cannot be factored.	(Solution 10) Step 1: Count the number of terms and variables $5x^3 - 2x^2 - 25x + 10$ has four terms and one variable x. Step 2: Arrangement Since $5x^3 - 2x^2 - 25x + 10$ has one variable, we arrange the expression in descending order. $5x^3 - 2x^2 - 25x + 10$is in descending order. Step 3: Grouping by dividing terms evenly. $5x^3 - 2x^2 - 25x + 10$has four terms, so two terms in each group.$(5x^3 - 2x^2) - (25x - 10)$ Step 4: Factoring out GCF from each group $(5x^3 - 2x^2) - (25x - 10)$ $= x^2(\mathbf{5x - 2}) - 5(\mathbf{5x - 2})$ $= (\mathbf{5x - 2})(x^2 - 5)$