Section 6.1 The Greatest Common Factor and Factoring by Grouping

Greatest Common Factor (GCF):

- The GCF is an expression of the highest degree that divides each term of the polynomial.
- The variable part of the greatest common factor always contains **the smallest power** of a variable that appears in all terms of the polynomial.

Finding GCF

- Step 1: Find the prime factorization of all integers and integer coefficients
- Step 2: List all the factors that are common to all terms, including variables
- Step 3: Choose the smallest power for each factor that is common to all terms
- Step 4: Multiply these powers to find the GCF
  NOTE: If there is no common prime factor or variable, then the GCF is 1

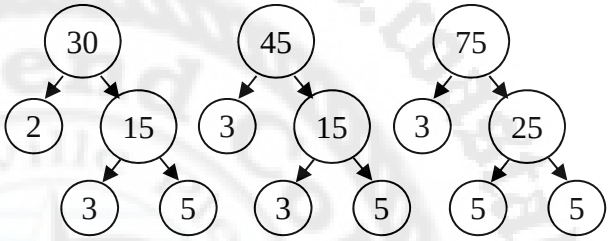
(Example) Find the GCF for the following set of algebraic terms: $\{30x^4y, 45x^3y, 75x^5y^2\}$

$30x^4y = 2\cdot\mathbf{3}\cdot5\cdot x\cdot x\cdot x\cdot x\cdot\mathbf{y} = 2\cdot\mathbf{3}\cdot5\cdot x^4\mathbf{y}$

$45x^3y = \mathbf{3}\cdot3\cdot5\cdot x\cdot x\cdot x\cdot\mathbf{y} = 3^2\cdot5\cdot x^3\mathbf{y}$

$75x^5y^2 = \mathbf{3}\cdot5\cdot5\cdot x\cdot x\cdot x\cdot x\cdot x\cdot\mathbf{y}\cdot\mathbf{y} = 3\cdot5^2\cdot x^5\mathbf{y}^2$

Therefore, GCF is $3\cdot5\cdot x^3\cdot y = 15x^3y$



¿Negative GCF?

- In my opinion, we do not need to factor out a negative GCF
- Factoring out a negative GCF is exactly same as factoring out GCF, but last step.
  - Step 1: Find the prime factorization of all integers and integer coefficients
  - Step 2: List all the factors that are common to all terms, including variables
  - Step 3: Choose the smallest power of each factor that is common to all terms
  - Step 4: Multiply these powers to find the GCF
  - Step 5: Put negative sign in front of the GCF

Factoring by Grouping will be discussed with examples

Exercises

Find the greatest common factor of the following list of terms. $18y,\ 8y^4,$ and $-10y^5$ The greatest common factor is ☐.	(Solution 1) Step 1: Prime Factorization (ignore signs) $18y = \mathbf{2}\cdot3\cdot3y = \mathbf{2}\cdot3^2\mathbf{y}$ $8y^4 = \mathbf{2}\cdot2\cdot2y^4 = \mathbf{2}^3\mathbf{y}^4$ $10y^5 = \mathbf{2}\cdot5y^5 = \mathbf{2}\cdot5\mathbf{y}^5$ Step 2: List all factors that are common to all terms Common factors to all are 2 and y Step 3: Choose the smallest power The smallest power for 2 is 1 The smallest power for y is 1 So, the GCF is $2^1\cdot y^1 = 2y$

Section 6.1 The Greatest Common Factor and Factoring by Grouping

Factor the polynomial using the greatest common factor.
If there is no common factor other than 1 and the polynomial cannot be factored, so state.

$6x^2 - 2x^4$

Select the correct choice below and fill in any answer boxes in your choice.

A. $6x^2 - 2x^4 =$ ☐

B. The polynomial cannot be factored.

(Solution 2)
Step 1: Prime Factorization (ignore signs)
$6x^2 = \mathbf{2} \cdot 3\boldsymbol{x}^2$
$2x^4 = \mathbf{2} \cdot \boldsymbol{x}^4$
Step 2: List all factors that are common to all terms
Common factors to all are **2** and ***x***
Step 3: Choose the smallest power
The smallest power for 2 is 1
The smallest power for *x* is 2
So, the GCF is $2^1 \cdot x^2 = \mathbf{2}\boldsymbol{x}^2$
Factoring out GCF
$6x^2 - 2x^4 = \mathbf{2}\boldsymbol{x}^2(\mathbf{3} - x^2)$
$\frac{6x^2}{2x^2} = 3, \quad \frac{-2x^4}{2x^2} = -x^2$

Factor the polynomial using the greatest common factor.
If there is no common factor other than 1 and the polynomial cannot be factored, so state.

$7x^4 - 21x^3 + 35x^2$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $7x^4 - 21x^3 + 35x^2 =$ ☐ (Factor completely.)

B. The polynomial cannot be factored.

(Solution 3)
Step 1: Prime Factorization (ignore signs)
$7x^4 = \mathbf{7}\boldsymbol{x}^4$
$21x^3 = 3 \cdot \mathbf{7}\boldsymbol{x}^3$
$35x^2 = 5 \cdot \mathbf{7}\boldsymbol{x}^2$
Step 2: List all factors that are common to all terms
Common factors to all are **7** and ***x***.
Step 3: Choose the smallest power
The smallest power for 7 is 1
The smallest power for *x* is 2
So, the GCF is $7^1 \cdot x^2 = \mathbf{7}\boldsymbol{x}^2$
Factoring out GCF
$7x^4 - 21x^3 + 35x^2 = \mathbf{7}\boldsymbol{x}^2(\boldsymbol{x}^2 - \mathbf{3}\boldsymbol{x} + \mathbf{5})$
$\frac{7x^4}{7x^2} = \boldsymbol{x}^2, \frac{-21x^3}{7x^2} = -\mathbf{3}\boldsymbol{x}, \frac{35x^2}{7x^2} = \mathbf{5}$

Factor the polynomial using the greatest common factor.
If there is no common factor other than 1 and the polynomial cannot be factored, so state.

$27x^2 - 29$

Select the correct choice below and fill in any answer boxes within your choice.

A. $27x^2 - 29 =$ ☐

B. The polynomial cannot be factored.

(Solution 4)
Step 1: Prime Factorization (ignore signs)
$27x^2 = 3 \cdot 3 \cdot 3x^2 = 3^3x^2$
$29 = 29$ because 29 is prime number
Step 2: List all factors that are common to all terms
There is **no common factor**.
Therefore, $27x^2 - 29$ cannot be factored.
or $27x^2 - 29$ is prime.

Section 6.1 The Greatest Common Factor and Factoring by Grouping

Factor the following polynomial using the negative of the greatest common factor.

$-4x^4+24x^3+20x^2$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○A. $-4x^4+24x^3+20x^2=$ ☐

○B. The polynomial cannot be factored.

(Solution 5)

Step 1: Prime Factorization (ignore signs)

$4x^4 = \mathbf{2}\cdot \mathbf{2}x^4 = \mathbf{2}^2x^4$

$24x^3 = \mathbf{2}\cdot \mathbf{2}\cdot 2\cdot 3x^3 = \mathbf{2}^3\cdot 3x^3$

$20x^2 = \mathbf{2}\cdot \mathbf{2}\cdot 5x^2 = \mathbf{2}^2\cdot 5x^2$

Step 2: List all factors that are common to all terms

Common factors to all are **2** and x.

Step 3: Choose the smallest power

The smallest power for 2 is 2

The smallest power for x is 2

GCF= $2^2\cdot x^2 = 4x^2 \Longrightarrow$ the negative GCF is $\mathbf{-4x^2}$

Factoring out the negative GCF

$-4x^4+24x^3+20x^2 = \mathbf{-4x^2}(\mathbf{x^2-6x-5})$

$\frac{-4x^4}{-4x^2}=\mathbf{x^2}$, $\frac{24x^3}{-4x^2}=\mathbf{-6x}$, $\frac{20x^2}{-4x^2}=\mathbf{-5}$

Factor the following polynomial using the negative of the greatest common factor.

$-12a^3b^2+16ab$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○A. $-12a^3b^2+16ab=$ ☐

○B. The polynomial cannot be factored.

(Solution 6)

Step 1: Prime Factorization (ignore signs)

$-12a^3b^2 = \mathbf{2}\cdot \mathbf{2}\cdot 3\mathbf{a}^3\mathbf{b}^2 = \mathbf{2}^2\cdot 3\mathbf{a}^3\mathbf{b}^2$

$16ab = \mathbf{2}\cdot \mathbf{2}\cdot 2\cdot 2\mathbf{ab} = \mathbf{2}^4\mathbf{ab}$

Step 2: List all factors that are common to all terms

Common factors to all are 2, a, and b.

Step 3: Choose the smallest power

The smallest power for 2 is 2

The smallest power for a is 1

The smallest power for b is 1

GCF= $2^2ab = 4ab \Longrightarrow$ the negative GCF is $\mathbf{-4ab}$

Factoring out the negative GCF

$-12a^3b^2+16ab = \mathbf{-4ab}(\mathbf{3a^2b-4})$

$\frac{-12a^3b^2}{-4ab}=\mathbf{3a^2b}$, $\frac{16ab}{-4ab}=\mathbf{-4}$

Factor the following polynomial using the greatest common binomial factor.

$12x(x+y)-(x+y)$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○A. $12x(x+y)-(x+y)=$ ☐ (Factor completely.)

○B. The polynomial cannot be factored.

(Solution 7)

Step 1: Prime Factorization (ignore signs)

$12x(x+y) = 2\cdot 2\cdot 3x(x+y) = 2^2\cdot 3x\mathbf{(x+y)}$

$(x+y) = \mathbf{(x+y)}$ since $(x+y)$ is prime

Step 2: List all factors that are common to all terms

Common factors to all are $(x+y)$.

Step 3: Choose the smallest power

The smallest power for $(x+y)$ is 1

So, the GCF is $\mathbf{(x+y)}$

Factoring out GCF

$12x(x+y)-(x+y) = \mathbf{(x+y)}(\mathbf{12x-1})$

$\frac{12x(x+y)}{x+y}=\mathbf{12x}$; $\frac{-(x+y)}{x+y}=\mathbf{-1}$

Factor the following expression by grouping.

$x^2+8x+9x+72$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $x^2+8x+9x+72=$ ☐ (Factor completely.)

B. The polynomial cannot be factored.

(Solution 8)

Step 1: Count the number of terms

$x^2+8x+9x+72$ has four terms. Thus factoring by grouping.

Step 2: Arrangement

Since $x^2+8x+9x+72$ has one variable, we arrange the expression in descending order.

$x^2+8x+9x+72$ is in the descending order.

Step 3: Grouping by dividing terms evenly.

$x^2+8x+9x+72$ has four terms, so two terms in each group. $(x^2+8x)+(9x+72)$

Step 4: Factoring out GCF from each group

$(x^2+8x)+(9x+72)$

$=x(x+8)+9(x+8)$

$=(x+8)(x+9)$

Factor the following expression by grouping.

$x^3-7x^2+9x-63$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $x^3-7x^2+9x-63=$ ☐ (Factor completely.)

B. The polynomial cannot be factored.

(Solution 9)

Step 1: Count the number of terms.

$x^3-7x^2+9x-63$ has four terms. Thus, factoring by grouping.

Step 2: Arrangement

Since $x^3-7x^2+9x-63$ has one variable, we arrange the expression in descending order.

$x^3-7x^2+9x-63$ is in descending order.

Step 3: Grouping by dividing terms evenly.

$x^3-7x^2+9x-63$ has four terms, so two terms in each group. $(x^3-7x^2)+(9x-63)$

Step 4: Factoring out GCF from each group

$(x^3-7x^2)+(9x-63)$

$=x^2(x-7)+9(x-7)$

$=(x-7)(x^2+9)$

Factor the following expression by grouping.

$5x^3-2x^2-25x+10$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $5x^3-2x^2-25x+10=$ ☐ (Factor completely.)

B. The polynomial cannot be factored.

(Solution 10)

Step 1: Count the number of terms.

$5x^3-2x^2-25x+10$ has four terms. Thus, factoring by grouping.

Step 2: Arrangement

Since $5x^3-2x^2-25x+10$ has one variable, we arrange the expression in descending order.

$5x^3-2x^2-25x+10$is in descending order.

Step 3: Grouping by dividing terms evenly.

$5x^3-2x^2-25x+10$has four terms, so two terms in each group.$(5x^3-2x^2)-(25x-10)$

Step 4: Factoring out GCF from each group

$(5x^3-2x^2)-(25x-10)$

$=x^2(5x-2)-5(5x-2)$

$=(5x-2)(x^2-5)$